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Article VII; it is proposed for action at the annual meeting¹ under the provisions of By-Law 1 of Article VIII.

W. D. CAIRNS, *Secretary-Treasurer*.

THE SIXTH SUMMER MEETING OF THE ASSOCIATION.

The sixth summer meeting of the Mathematical Association of America was held by invitation at Wellesley College on Tuesday, Wednesday and Thursday, September 6-8, 1921, in conjunction with, and immediately preceding, the summer meeting of the American Mathematical Society. 146 were present at the meeting, including the following 106 members of the Association:

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| C. R. ADAMS, Harvard University. | ARCHIBALD HENDERSON, University of North Carolina. |
| R. C. ARCHIBALD, Brown University. | GOLDIE P. HORTON, University of Texas. |
| CLARA L. BACON, Goucher College. | L. A. HOWLAND, Wesleyan University. |
| I. A. BARNETT, University of Saskatchewan. | L. S. HULBURT, Johns Hopkins University. |
| IDA BARNEY, New Haven, Conn. | E. V. HUNTINGTON, Harvard University. |
| SUZAN R. BENEDICT, Smith College. | W. A. HURWITZ, Cornell University. |
| A. A. BENNETT, University of Texas. | NELLE L. INGELS, Washington, D. C. |
| E. G. BILL, Dartmouth College. | DUNHAM JACKSON, University of Minnesota. |
| G. D. BIRKHOFF, Harvard University. | O. D. KELLOGG, Harvard University. |
| VEVIA BLAIR, Horace Mann School. | J. P. KELLY, Boston College. |
| G. A. BLISS, University of Chicago. | W. D. LAMBERT, Coast and Geodetic Survey. |
| L. A. BRIGHAM, Boston University. | MARCIA L. LATHAM, Hunter College. |
| B. H. BROWN, Harvard University. | ANNA D. LEWIS, Lake Erie College. |
| R. E. BRUCE, Boston University. | FLORENCE P. LEWIS, Goucher College. |
| W. D. CAIRNS, Oberlin College. | W. R. LONGLEY, Yale University. |
| B. H. CAMP, Wesleyan University. | A. C. LUNN, University of Chicago. |
| F. E. CARR, Oberlin College. | E. S. MANSON, Ohio State University. |
| A. B. CHACE, Brown University. | R. M. MATHEWS, Wesleyan University. |
| J. W. CLAWSON, Ursinus College. | NANNIE J. MCKNIGHT, Western High School, Washington, D. C. |
| G. M. CONWELL, N. Y. State College for Teachers. | T. E. MERGENDAHL, Tufts College. |
| J. L. COOLIDGE, Harvard University. | HELEN A. MERRILL, Wellesley College. |
| LENNIE P. COPELAND, Wellesley College. | A. L. MILLER, Brookline, Mass. |
| C. H. CURRIER, Brown University. | G. A. MILLER, University of Illinois. |
| C. E. DIMICK, U. S. Coast Guard Academy. | E. B. MODE, Boston University. |
| ELEANOR C. DOAK, Mount Holyoke College. | C. N. MOORE, University of Cincinnati. |
| W. P. DUFEE, Hobart College. | R. K. MORLEY, Worcester Polytechnic Institute. |
| J. A. EIESLAND, West Virginia University. | H. C. M. MORSE, Cornell University. |
| L. P. EISENHART, Princeton University. | J. R. MUSSELMAN, Johns Hopkins University. |
| H. J. ETTLINGER, University of Texas. | C. E. NORWOOD, U. S. Naval Academy. |
| J. A. FOBERG, Penna. Department of Public Instruction. | W. F. OSGOOD, Harvard University. |
| T. C. FRY, Western Electric Company. | ANNA H. PALMIÉ, Western Reserve University. |
| C. A. GARABEDIAN, Harvard University. | ANNA J. PELL, Bryn Mawr College. |
| H. D. GAYLORD, Browne and Nichols School. | L. R. PERKINS, Middlebury College. |
| D. C. GILLESPIE, Cornell University. | E. C. PHILLIPS, Woodstock College. |
| R. E. GILMAN, Brown University. | SUSAN M. RAMBO, Smith College. |
| O. E. GLENN, University of Pennsylvania. | W. R. RANSOM, Tufts College. |
| C. F. GUMMER, Queen's University. | C. N. REYNOLDS, West Virginia University. |
| E. R. HEDRICK, University of Missouri. | |

¹ The unavoidable delay in the appearance of the October issue made it impossible to give the necessary notice, in the MONTHLY, one month before the annual meeting; hence it was not possible to adopt this amendment at the annual meeting in December, 1921.

HARRIS RICE, Worcester Polytechnic Institute.
 L. H. RICE, Mass. Institute of Technology.
 R. G. D. RICHARDSON, Brown University.
 E. D. ROE, Jr., Syracuse University.
 GEORGE RUTLEDGE, Mass. Institute of Technology.
 F. H. SAFFORD, University of Pennsylvania.
 HAZEL E. SCHOONMAKER, Gulf Park College.
 W. G. SIMON, Western Reserve University.
 H. L. SLOBIN, New Hampshire State College.
 CLARA E. SMITH, Wellesley College.
 D. E. SMITH, Columbia University.
 P. F. SMITH, Yale University.
 MAY J. SPERRY, Knox College.
 J. A. TOBIN, Boston College.
 M. O. TRIPP, Wittenberg College.
 H. W. TYLER, Mass. Institute of Technology.

ROXANA H. VIVIAN, Wellesley College.
 J. L. WALSH, Harvard University.
 A. G. WEBSTER, Clark University.
 J. K. WHITTEMORE, Yale University.
 RUBY WILLIS, Wellesley College.
 EUPHEMIA R. WORTHINGTON, Wellesley, Mass.
 RUTH G. WOOD, Smith College.
 F. S. WOODS, Mass. Institute of Technology.
 FRANCES W. WRIGHT, Elmira College.
 C. H. YEATON, Oberlin College.
 JESSICA M. YOUNG, Washington University.
 J. W. YOUNG, Dartmouth College.
 MABEL M. YOUNG, Wellesley College.
 S. D. ZELDIN, Mass. Institute of Technology.
 NOTRE DAME UNIVERSITY, Joseph Donahue,
 official representative.

Among those who came from more distant parts of the country there were in attendance three from Canada, three from Texas, two from Missouri and one each from Minnesota, Mississippi, and North Carolina.

The committee on local arrangements planned most effectively for the comfort of guests and this was recognized in a motion presented by Professor C. N. Moore and heartily adopted in the Thursday morning session. Well-appointed quarters were provided in Tower Court, one of the magnificent dormitories, the members using this building also for their meals, for committee meetings and for frequent social gatherings. On Tuesday afternoon an opportunity was afforded to visit the private gardens of the beautiful Hunnewell estate; in the evening an organ recital was given by Professor Macdougall of Wellesley College, and parties were afterward taken on a visit to Whitin Observatory. On Thursday afternoon a visit was made to the unusual collection of butterflies belonging to the Denton brothers in Wellesley. After the close of the Society program Friday noon, automobiles were placed at the disposal of all who wished to remain, for a ride to the historic spots in Lexington and Concord.

The joint banquet of the Society and the Association was attended by about 120 members and visitors. President Ellen F. Pendleton gave a gracious welcome to Wellesley's guests, and under the toastmastership of President Bliss of the Society speeches were made by President Miller of the Association, Professors Huntington, Jackson, Hedrick, Lewis, Eisenhart and Webster.

GIFT TO THE ASSOCIATION FOR THE PUBLICATION OF MATHEMATICAL MONOGRAPHS.

The outstanding feature of the Wellesley meeting was the announcement of a notable gift made to the Association. The nature and the terms of the gift cannot be presented to our members and to the mathematical fraternity more clearly than in the admirable letter of transmission from the donor:

LA SALLE, ILL., August 24, 1921.

PROFESSOR H. E. SLAUGHT,
Chicago, Illinois.

Dear Professor Slaughter:

Confirming our recent conferences regarding the publication of a series of monographs on mathematical subjects, to be prepared and published by The Mathematical Association of America, I as trustee for the Edward C. Hegeler Trust Fund hereby offer a yearly contribution to the Association of twelve hundred dollars (\$1200) for a period of five years payable on January first of each year and beginning in January 1922, these funds to be under the complete control of The Mathematical Association of America. It is understood that the distribution of these monographs to the members of the Association will naturally be conducted through its own channels, but for their distribution to the general public the Open Court Publishing Company gladly offers its services.

If at the end of five years this project shall have proved successful it is my intention to then give to the Association a permanent endowment fund, and I will so direct my legal representatives, which will yield at least twelve hundred dollars annually, hoping that this will be sufficient income for the publication of two or three monographs a year, as long as this form of publication appears desirable and best suited to the diffusion of mathematical and formal thought as contributory to exact knowledge and clear thinking, not only for mathematicians and teachers of mathematics but also for other scientists and the public at large. But nothing is hereby intended to restrain the Association from the use of this income for the promotion of other mathematical publications; if at some future time it should be deemed best to cease the publication of such monographs.

The Open Court Publishing Company has also been working for the diffusion of mathematical knowledge through a related channel, namely, by making certain mathematical classics available to the public through reprints, and in the furtherance of this ideal it is hoped that its coöperation with the Association will prove desirable and mutually helpful, but it is not intended to bind either organization.

It is my hope that The Mathematical Association of America will be successful in editing such a series and that it will set a standard of simplicity and clearness not before attained by mathematical writers in English.

Trusting that this donation may be of general benefit, I am

Yours very truly,

(Signed) MARY HEGELER CARUS,

Trustee Edward C. Hegeler Trust Fund.

Appropriate action has been taken by the Trustees in formally accepting this gift and expressing their grateful recognition of this notable opportunity for an advance in American mathematics. Further, by a motion adopted unanimously in the morning session of the Association, Professor Smith and the Secretary were instructed to send a letter to Mrs. Carus embodying the Association's keen appreciation of her generous gift. This letter follows.

Dear Mrs. Carus:

At a meeting of the Mathematical Association of America, held at Wellesley College September 7, 1921, there was read to the members your letter of August 24, 1921, announcing the notable gift made by you for the purpose of enabling the Association to publish a series of mathematical monographs. Although it was further announced that the Trustees had formally accepted the trust, together with the obligations attendant thereon, and that they had expressed to you their appreciation of your generosity, the Association, desiring also to express its own sentiments upon this memorable occasion, passed the following resolution and delegated the undersigned to convey the same by letter to you:

Resolved that the Mathematical Association of America, having been advised of the munificent gift of Mrs. MARY HEGELER CARUS as Trustee of the Edward C. Hegeler Trust Fund, desires to express its sincere appreciation of her action and to assure her of its determination to make every effort to carry out the trust in accord with the desires which she has so clearly set forth. The Association joins with her in the expression of the belief that it is both possible and feasible to

present to a large body of intelligent readers a clear and satisfying view of the leading branches of modern mathematics which, under present conditions, they are unable to obtain. To this end the Association pledges its efforts to secure an efficient board of editors who will carry out the project, mindful of its responsibilities and determined to make of the series the success for which we all confidently hope.

Yours very truly,

DAVID EUGENE SMITH,
W. D. CAIRNS.

President Miller presided at the sessions of the Association, save that Vice-President Archibald replaced him for a part of the Wednesday morning session. Presidents Bliss and Miller occupied the chair at the joint sessions of Wednesday afternoon and Thursday morning respectively. The program was formulated under the chairmanship of Professor W. R. Longley and the committee's able rendering of its duties was commended in Professor Moore's motion. When it was announced at the final session that Professor A. A. Bennett had been appointed Editor-in-Chief of the *AMERICAN MATHEMATICAL MONTHLY* to succeed Professor Archibald, whose resignation had been accepted by the Trustees to take effect at the close of the present volume, the following resolution was offered by Professor Jackson and heartily adopted by the Association:

"Voted, That the Association place on record its appreciation of the faithful, enthusiastic, and scholarly services of Professor Archibald as Editor-in-Chief of the *MONTHLY*, constituting a contribution to its history which no other man could have made and a significant stage in its advancement toward a permanent place among the most important mathematical journals."

The following papers were read, the last two being given in the joint session of the Society and the Association.

(1) "Freshman Mathematics" by Professor W. F. OSGOOD, Harvard University.

General discussion, led by Dean E. G. BILL, Dartmouth College, and Professor CLARA L. BACON, Goucher College.

(2) Report of progress of the National Committee on Mathematical Requirements, by Professor J. W. YOUNG, Dartmouth College.

(3) "Religio mathematici"—retiring presidential address by Professor D. E. SMITH, Teachers College, Columbia University.

(4) "Synthetic projective methods of generating cubic and quartic curves" by Professor HELEN A. MERRILL, Wellesley College.

(5) "A few questionable points in the history of mathematics" by Professor G. A. MILLER, University of Illinois.

(6) "A simple form of Duhamel's theorem and some new applications" by Professor H. J. ETTLINGER, University of Texas.

(7) "Some mathematical aspects of the theory of relativity" by Professor JAMES PIERPONT, Yale University.

(8) "The place of the Einstein theory in theoretical physics" by Professor A. C. LUNN, University of Chicago.

Abstracts of the papers and discussions follow below, the numbers corresponding to the numbers in the list of titles.

1. Calculus is now rather generally accepted as an essential part of the curriculum of the freshman year, the more readily for the fact that the influential class of technical schools require trigonometry and solid geometry for entrance. A generation ago a leisurely treatment of calculus was given in the sophomore year, largely formal as represented in the texts of that period, the applications being for the most part geometrical and very closely following the prototype. Next followed what may be called the Perry movement. Professor Osgood feels it imperative to counsel a middle course. Numerous important principles were presented in his mapping out of the freshman course. The applications of the differential calculus to maxima and minima and allied topics should be included in a first course, and here as in later courses mathematicians must lay hold of the definite integral as the limit of a sum as an illuminating feature of the calculus. Formal work is to be carried to the point that for the easier applications the student need not be held up by the necessary differentiation or integration.

As to the amount of work that can be included, the speaker noted that in S. A. T. C. days he found that in one third of a year, in about thirty periods, it was possible to take up the vital parts of analytic geometry, including some applications, for example, the geometrical determination of points on an ellipse, etc. It was not a course for *seeing* analytic geometry; nevertheless it transformed the freshman from a high school boy into a college student. Then it was possible to begin the calculus.

Early in this course it is necessary for the pupil to see the derivative at work; the question of what the derivative is can be gained by seeing what it is for. The place for introducing the infinitesimal depends on the individual, Professor Osgood finding it convenient to put it at the end of the first group of differentiations before transcendental functions, this with full recognition of the fact that college teachers should not feel forced to turn out a finished product in their classes. For example, briefly stated, we may begin with the familiar definition $\lim_{\Delta x \rightarrow 0} \Delta y / \Delta x = D_x y$, and since $\Delta y / \Delta x = D_x y + \epsilon$, ϵ being a correction term, we have as a consequence $\Delta y = D_x y \cdot \Delta x + \epsilon \Delta x$, whence we get $dy = D_x y \cdot \Delta x$. Then we prove that $dy = D_x y \cdot dx$, etc.

Next comes the differentiation of trigonometric functions, with the addition of which there is ample material for drill in the formal side of differentiation. We may now introduce additional problems in maxima and minima involving trigonometric functions; here is a natural place to treat the analytical part of trigonometry.

The student should now be given the great advantage which the calculus affords in curve tracing. The differentiation of logarithmic functions comes next. Inverse functions need not enter until the work on integration.

But here the new mathematics diverges sharply from the old. We must now get over the subject of indefinite integrals, treating the necessary definitions and

formulas, for example, the clear realization of the fact that these equations all have the same meaning:

$$U = \int u dx; \quad D_x U = u; \quad dU = u dx.$$

The several elementary principles of differentiation (derivative of a sum, etc.) must needs be given in this course as much as in the older course. The number of formulas can however be limited to less than twelve, the others being deduced by several methods, the most important of which are *gumption* and substitution.

Professor Osgood then introduces simple problems in physics, explaining the method of defining elementary work for a variable force, in extension of the pupil's high school principle of work, and thus leading up to the use of Duhamel's theorem. He closed his address by saying that there never has been a time when teachers were more at a premium than today; that teachers need continually to adapt their methods to the various conditions, for example, that women students are not as a class vitally interested in the problems of physics; that there should be inculcated in our pupils as fully as possible: (1) a broad and deep interest in mathematical analysis for its own sake, (2) a sympathetic attitude toward the outstanding problems of physics, (3) judgment on the scientific side of choosing that which is genuine and fundamental and of avoiding that which is impossible of attainment, and also, judgment as to the proper presentation of this material.

Dean Bill stated that the calculus is best adapted to present those methods that are fundamental. The pupil should (1) acquire the ability to analyze a problem, (2) have proper tools, (3) seek the one prime requisite of accuracy, the average teacher being apt to pass a student on the ground that he has the right idea, even though he be woefully deficient in expressing this idea accurately. To accomplish this, (1) we must start the student in a correct manner from the outset, it being a psychological error to begin with mathematical reviews rather than to stimulate the student by taking up topics that are at least ostensibly new; (2) we must recognize that the pupil, like ourselves, cannot see many steps in advance and that it is by far better to master one subject fully than to cover many subjects and so to contribute to the great American trait of superficiality. It may be on this last ground that one may criticize Professor Osgood's course for trying to cover too much in the first short calculus course; it may be possible however to cut down the material in trigonometry and in conic sections sufficiently to gain time for the calculus.

Miss Bacon remarked that Professor Osgood's course as outlined was stimulating for those who have presented advanced entrance credits or are specially gifted in mathematics, but that it is difficult to do justice to trigonometry and analytic geometry and to bring all of this material into the second semester, particularly in a three hour course. She advocated having two freshman courses, one similar to that suggested by Professor Osgood, consisting of analytic geometry in the first semester and calculus in the second, the other for the ordinary freshman, less mature mathematically, who offers only plane trigonometry and elementary algebra, the course to consist of trigonometry for the first semester and analytic geometry for the second, leaving the calculus for the sophomore year.

Professor Longley spoke of the great variation in the notion of freshman mathematics, as indicated by the results of a questionnaire sent to about thirty colleges and universities from Yale; for example, the amount of time devoted to advanced algebra varied from zero to fifty-five hours, trigonometry varied greatly, conics varied from zero to twenty-four lessons. Only a few of these taught as much as a half year of freshman calculus. The reports differed so much from each other that no general conclusions could be derived from the data.

Professor Glenn expressed his pleasure at hearing of freshman teaching based on genuine theory rather than pedagogical convenience and observed that something may still perhaps be said as to giving a freshman more in some parts of his first course than he can stand.

In reference to students who had not had trigonometry, Professor Osgood said that while the numerical portion of trigonometry should not be curtailed, the analytical portion can almost all be treated in connection with calculus; that, for example, trigonometry might be taught for five weeks, such topics as the addition theorem being touched upon in the first treatment, the remainder of the first half year reserved for analytic geometry, thus using the second half year for calculus with a fuller treatment of some topics of the first half year.

Professor Ransom, in commenting on Professor Longley's remarks, felt sure that the general tendency of colleges over the country is toward two types: one the old-fashioned type consisting of extracts from college algebra, solid geometry and analytic geometry, the other, now emerging, including parts of trigonometry, analytic geometry and calculus.

2. The National Committee held its last meeting under its present form of organization on September fifth. One phase, at least, of its work has come to an end. In view of several previous reports made to the Association, it hardly seems necessary at this time to review in detail the work of the Committee. The present report may well be confined to a statement regarding present conditions and plans for the immediate future. The manuscript of a summary of the final report of the Committee has been sent to the U. S. Bureau of Education for publication. This summary, which will constitute a bulletin of some eighty pages, virtually presents the first part of the complete report. It contains the following chapters:

- I. A Brief Survey of the Report
- II. Aims of Mathematical Instruction—General Principles
- III. Mathematics for Years Seven, Eight and Nine
- IV. Mathematics for Years Ten, Eleven and Twelve
- V. College Entrance Requirements in Mathematics
- VI. List of Propositions in Plane and Solid Geometry
- VII. The Function Concept in Elementary Mathematics
- VIII. Terms and Symbols in Elementary Mathematics,

and also a brief synopsis of the remaining chapters of the complete report. It is expected that this summary will appear late in November or early in December.

It was the original intention of the Committee to publish its complete report

also through the U. S. Bureau of Education. It was found, however, that this would involve a delay of two or three years in view of the fact that it would have been necessary for the Bureau of Education to issue the report in parts extending over a considerable period of time. It is hoped at present that sufficient funds will be obtainable to publish the report during the winter and to distribute it free of charge to all who are sufficiently interested to ask for it. The complete report will constitute a volume of about five thousand pages. In addition to the chapters listed in the summary, it will contain an account of a number of investigations instituted by the Committee. Among these may be mentioned:

The Present Status of Disciplinary Values in Education
A Critical Study of the Correlation Method Applied to Grades
Mathematical Curricula in Foreign Countries
Mathematics in Experimental Schools
The Use of Mental Tests in the Teaching of Mathematics
The Training of Teachers of Mathematics.

There will also be included an extensive bibliography on the teaching of mathematics.

In closing this phase of its work, the Committee desires to extend its most cordial thanks to all the individuals and organizations that have helped. The response secured by the Committee to its appeal for assistance in solving the many problems facing it has been extremely enthusiastic and gratifying. This leads the Committee to look forward to the future optimistically. The real work for which the Committee was appointed may be said to begin with the publication of its report rather than to end with it. Continued enthusiastic activity on the part of all individuals and organizations concerned with the teaching of mathematics is needed over a period of many years to put the recommendations of the Committee into effect, to test their validity and to modify them in ways that experience shows to be desirable. In order to be of assistance in this direction, the Committee hopes to be able to maintain an office with a certain amount of clerical help during the next few years so that it may continue to act as a clearing house for ideas and to stimulate the discussion of problems relating to the teaching of mathematics among the nearly one hundred organizations that have in the past been actively coöperating with the Committee. Members of this Association are strongly urged to continue their support and activity in connection with the cause for which the National Committee has been laboring.

Following Professor Young's report the session adopted by a hearty vote a motion of Professor Coolidge recognizing the great importance of the series of reports, their potential influence for good in the cause of mathematics if these reports come fully to the attention of American teachers, and commending the committee members and those collaborating with them for the high order of ability and thoroughness shown in these reports.

3. President Smith's able retiring address is published elsewhere in this issue of the MONTHLY.

4. The methods ordinarily given in the texts for the generation of cubics by two flat pencils of the first and second order respectively can be greatly shortened by substituting for one or both of these pencils a flat pencil of the first order in involution. Schröter has proved that the general cubic may be generated by two such pencils related projectively and having a common corresponding ray, while, if the common ray is not corresponding, a quartic is generated. Unicursal cubics may be generated by relating projectively two flat pencils of the first order, one of which is in involution, and then special cubic curves may be obtained very readily from systems of coaxial circles.

Miss Merrill's paper, embodying a method which has proved especially interesting even to first year students in two or three colleges, and which gives points on the curves very expeditiously, since these ordinarily are given in sets of four, may be expected in an early number of the MONTHLY.

5. Professor Miller's paper, in effect, appeared in *Scientific Monthly*, September, 1921.

6. Professor Ettlinger presented a very simple form of Duhamel's theorem by means of the following geometric lemma: Let the interval $I: a \leq x \leq b$ be divided into n equal subdivisions, I_{in} , of length, $\Delta x_n = (b - a)/n$, ($i = 1, 2, \dots, n$). Upon I_{in} as base, draw a rectangle of area, R_{in} , and height h_{in} , such that $|h_{in}| \leq M$ for all values of n . If $\bar{P}$ is any fixed point of I , there is for each value of n , at least, one rectangle whose base contains $\bar{P}$. If for each fixed $\bar{P}$, the altitude of this rectangle, h_P , approaches zero as $n \rightarrow \infty$, then $\lim_{n \rightarrow \infty} \sum_{i=1}^n R_{in} = 0$.

The advantages of this form are fourfold: (1) it is simple enough for a first course in the calculus, (2) it avoids the question of double limits and uniform convergence, (3) it is more general than earlier contributions, (4) it is easily applied.

A simple proof of the integrability of a continuous function is given by means of this formulation. Finally, new applications are made to the transformation of a double integral and to the solution of an integral equation.

The discussion of this paper turned rather upon the wisdom of using Duhamel's theorem or a substitute; it was observed however that Professor Ettlinger apparently avoided the use of uniform convergence, but that logically this principle must enter into any proof of Duhamel's theorem or its equivalent.

In his inimitable manner Professor Huntington laid, as he said, a wreath on the tomb of Duhamel's theorem for the very good reason that he believed it to be dead, had great respect for it and had a belief in its immortality. He illustrated how one can set up a definite integral with perfect rigor without the use of Duhamel's theorem, his full presentation having been given in this MONTHLY, 1917, 271-275.

Professor Bliss discussed the problem of the total attraction due to a thin rod, using the substitute for Duhamel's theorem which he published in the *Annals of Mathematics* for December, 1914.

Professor Birkhoff mentioned his increasing attachment to Duhamel's theorem. He has found such an illustration as this illuminating and easily understood: If I am calculating the cost of building from the estimates of various sub-contractors, each being correct to within 10 per cent., I am certain that my whole estimate is correct to within 10 per cent. He considers it wise to use, not the more difficult problem of attraction, but a simpler application, the length of an arc. After simple examples he tells his students that it is well for them to read Duhamel's theorem, his experience being that a gratifying number of students will work their problems by actually using Duhamel's theorem.

7. Professor Pierpont's purpose, achieved in a wonderfully clear manner, was to present in simple form the main mathematical facts on which Einstein's theory rests. As an extension of two- and three-dimensional concepts he described an n -way space or manifold, a hypersurface, distance and its connection with the theory of bilinear forms, angle, volume, geodetic curve, and the term "curvature of space," which is the generalization of the Gaussian curvature of a surface with its invariant $1/(R_1 R_2)$. After he explained the mathematical notion involved in tensors and their properties, he showed how by generalizing the vector treatment of Maxwell's equations and similar equations Einstein derived his fundamental equations for a universe in which the geometry of a space is determined not merely through the conventions as to length and time, but by all the material that exists in this space. He finally sketched the still more generalized theory of Weyl, wherein any existing amounts of electricity and magnetism serve also to determine the geometry.

It will be of great service to all interested in the Einstein theory if, as is expected, Professor Pierpont's very helpful introduction to the difficulties of the mathematics of relativity may be made available through an American journal.

8. Because of the nature of the subject it is difficult to outline Professor Lunn's presentation of the place of the Einstein theory in theoretical physics. He sought in the main to give the physical interpretation of the mathematical notions and principles involved in Einstein's theory, indicating for example how the notion of n dimensions may be realized experimentally. A considerable part of the address had to do with a description of the vector notation and expressions used in the development of the theory.

MEETING OF THE BOARD OF TRUSTEES OF THE ASSOCIATION.

Ten members of the Board were present at this meeting.

(a) The Trustees received the formal letter from Mrs. Mary Hegeler Carus already presented in this report. The Board voted formally to accept the gift and to instruct the Secretary to send to Mrs. Carus the acceptance and the expression of gratefulness with which the Trustees regard this gift. It was voted to express the obligation of the Board to Professor Slaughter for his large share in inspiring this gift and for his continued services to the Association in

this great enterprise. A committee, consisting of Professors Slaughter, Veblen and Cairns, was appointed to nominate to the Trustees an editorial board on monographs and to formulate a statement of the powers of this board.

(b) In view of the resignation of Professor Archibald as Editor-in-Chief of the MONTHLY, to take effect with the completion of the present volume for 1921, Professor A. A. BENNETT of the University of Texas was appointed as his successor.

(c) The following seventy persons and five institutions, on applications duly certified, were elected to membership:

To individual membership.

- L. C. BAGBY, A.B. (Washburn). Head of dept. of math., Ottawa Univ., Ottawa, Kans.
 A. J. BARRETT, A.B. (Arkansas). Instr., U. S. Naval Acad., Annapolis, Md.
 E. T. BROWNE, A.M. (Virginia). Asst. prof., Trinity Coll., Hartford, Conn.
 JULIA A. BURRELL, A.B. (Rice Inst.). Galveston, Tex.
 W. R. BURROWS, Medical student, Univ. of Chicago. Halstead, Kans.
 MILDRED E. CARLEN, Student, Brown Univ., Secretary to the Editor-in-Chief of the MONTHLY, Providence, R. I.
 E. L. CARR, A.B. (Ewing Coll.). Prof., Shurtleff Coll., Alton, Ill.
 MARGARET F. CHAPMAN, A.B. (Wisconsin). Instr., State Normal School, Milwaukee, Wis.
 E. F. CHURCH, C.E. (Syracuse). Prof., Penna. Military Coll., Chester, Pa.
 LENA R. COLE, A.M. (Missouri). Head of dept. of math., Central Coll., Lexington, Mo.
 ALBERTUS DARNELL, Ph.B. (Michigan). Head of dept. of math., Detroit Junior Coll., Detroit, Mich.
 H. A. DAVIS, Asst., W. Va. Univ., Morgantown, W. Va.
 A. K. DENNY, M.S. (Lincoln). Prof. and registrar, Lincoln Coll., Lincoln, Ill.
 L. E. DONNELLY, Student, Univ. of Chicago, Chicago, Ill.
 B. F. DOSTAL, A.M. (Indiana). Asst. prof., Univ. of Denver, Denver, Col.
 FRANC C. EARHART, B.S. (Lenox). Dean of Women, Lenox Coll., Hopkinton, Iowa.
 LOUIS FELDMAN, B.S. in E.E. (Cooper Union). Brooklyn, N. Y.
 J. C. FIELDS, Ph.D. (Johns Hopkins). Prof., Univ. of Toronto, Toronto, Canada.
 GLADYS H. FREEMAN, A.M. (Chicago). Instr., State Coll. of Washington, Pullman, Wash.
 T. C. FRY, Ph.D. (Wisconsin). Mathematician, Research Labs., Amer. Tel. and Tel. Co., and Western Electric Co., New York, N. Y.
 LESLIE J. GAYLORD, A.B. (Lake Erie). Instr., Agnes Scott Coll., Decatur, Ga.
 GLADYS E. C. GIBBENS, Ph.D. (Chicago). Instr., Univ. of Minn., Minneapolis, Minn.
 G. W. GORRELL, A.M. (Ohio State). Prof., Colo. School of Mines, Golden, Colo.
 R. F. GRAESSER, A.B. (Illinois). Asst., Univ. of Illinois, Urbana, Ill.
 LILIAN HACKNEY, A.B. (W. Va. Univ.). Head of dept. of math., Marshall Coll., Huntington, W. Va.
 W. R. HALE, A.M. (Alabama). Asst. prof., Colo. School of Mines, Golden, Colo.
 CLARA L. HANCOCK, A.M. (Iowa). Teacher, Junior Coll., Virginia, Minn.
 D. C. HARKINS, A.B. (W. Va. Univ.). Asst., W. Va. Univ., Morgantown, W. Va.
 C. E. HARRINGTON, M.E. (Cornell). Instr., Univ. of Buffalo, Buffalo, N. Y.
 MARY G. HASEMAN, Ph.D. (Bryn Mawr). Instr., Univ. of Illinois, Urbana, Ill.
 B. A. HAZELTINE, B.S. (Tufts). Instr., Tufts Coll., Tufts College, Mass.
 ARCHIBALD HENDERSON, Ph.D. (North Carolina; Chicago), D.C.L. (Univ. of the South). Head of dept. of math., Univ. of North Carolina, Chapel Hill, N. C.
 M. DET. HIGH, A.B. (Franklin & Marshall). Head of dept. of math. and dean of men, Central State Normal School, Lock Haven, Pa.
 T. R. HOLLCROFT, Ph.D. (Cornell). Prof., Wells Coll., Aurora, N. Y.
 F. F. HOOPER, A.M. (Wisconsin). Dean, Univ. of Chattanooga, Chattanooga, Tenn.
 E. M. HORSBURGH, D.Sc. (Edinburgh). Lecturer on technical math., Univ. of Edinburgh, Edinburgh, Scotland.
 MILDRED HUNT, A.M. (Chicago). Head of dept. of math., Bessie Tift Coll., Forsyth, Ga.
 K. P. JOHNSTON, B.A., B.S. (Queens). Asst. prof., Queens Univ., Kingston, Ont., Canada.

- AGNES A. JONES, A.B. (Hood Coll.). Grad. student, Univ. of Chicago. Larimer, Pa.
 E. E. JORDAN, M.A. (Dalhousie). Asst. prof., Univ. of Brit. Columbia, Point Grey, B. C., Canada.
 M. H. JÛRDAK, M.A. (Beirut). Prof., Amer. Univ. of Beirut, Beirut, Syria.
 J. P. KELLY, A.M. (Woodstock). Prof., Boston Coll., Chestnut Hill, Mass.
 J. E. KING, M.S. (Wisconsin). Prof., Iowa Wesleyan Coll., Mt. Pleasant, Iowa.
 C. G. LATIMER, B.S. (Chicago). Instr., Swarthmore Coll., Swarthmore, Pa.
 W. H. LYONS, A.M. (Denver). Muskegon Heights, Mich.
 C. C. MACDUFFEE, Ph.D. (Chicago). Instr., Princeton Univ., Princeton, N. J.
 H. B. MARSH, A.M. (Amherst). Head of dept. of math., Tech. High School and Junior Coll., Springfield, Mass.
 E. J. MAURUS, M.S. (Notre Dame). Prof., Univ. of Notre Dame, Notre Dame, Ind.
 W. C. MCCOY, A.B. (Ohio State). Columbus, Ohio.
 NANNIE J. MCKNIGHT, A.M. (George Washington). Teacher, Western High School, Washington, D. C.
 J. J. MILLER, B.L. (Ouachita). Head of dept. of math., Southeastern State Normal School, Durant, Okla.
 MRS. DECIMA E. MITCHELL, M.S. (Alberta). Instr., Univ. of Alberta, Edmonton, Alb., Canada.
 R. L. MODESITT, A.M. (Indiana). Teacher, Eastern Ill. State Teachers Coll., Charleston, Ill.
 W. A. MOORE, A.M. (Chicago). Prof., Birmingham-Southern Coll., Birmingham, Ala.
 C. M. NOLAND, A.B. (Missouri). Prof., Howard Payne Coll., Brownwood, Tex.
 J. O. OSBORN, A.B. (Berea). Instr., Cornell Univ., Ithaca, N. Y.
 XAVIER PRUM, A.M. (Louvain). Prof., Columbia Coll., Dubuque, Iowa.
 P. L. REA, A.B. (Oberlin). Instr., Marietta Coll., Marietta, Ohio.
 C. A. REAGAN, B.S. (Moores Hill). Prof., Hanover Coll., Hanover, Ind.
 SISTER MARY RESIGNATA, A.M. (Catholic Univ.). Head of dept. of math., Mount St. Joseph Coll., Dubuque, Iowa.
 ARVID REUTERDAHL. Dean, Dept. of engg. and archit., Coll. of St. Thomas, St. Paul, Minn.
 HAZEL E. SCHOONMAKER, A.M. (Radeliffe). In charge of math., Gulf-Park Coll., Gulfport, Miss.
 P. K. SMITH, A.M. (South Carolina). Adj. prof., Univ. of South Carolina, Columbia, S. C.
 R. W. THOMAS, B.S. (Washington & Jefferson). Instr., Washington & Jefferson Coll., Washington, Pa.
 TELESFORO TIENZO, A.B. (Philippines), B.S. (Chicago). Instr., Univ. of the Philippines, Manila, P. I.
 B. L. WAITS, A.M. (Clark). Prof. of math. and physics, Fla. A. & M. Coll., Tallahassee, Fla.
 W. W. WEBER, A.M. (Georgia). Prof. and dean, Southern Coll., Clearwater, Fla.
 R. L. WILDER, M.S. (Brown). Instr., Univ. of Texas, Austin, Tex.
 P. D. WILKINS, A.B. (Bowdoin). Instr., Tufts Coll., Tufts College, Mass.
 ARTHUR WOODS, M.A. (Queens). Instr., Western Univ., London, Ont., Canada.

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 VIRGINIA POLYTECHNIC INSTITUTE, Blacksburg, Va.
 COLLEGE OF WILLIAM AND MARY, Williamsburg, Va.

(d) The President was empowered to appoint a representative of the Association on the Council of the American Association for the Advancement of Science, in addition to the Secretary-Treasurer, who is already a member; President Miller has appointed to this position Professor H. L. RIETZ of the University of Iowa.

(e) Professors A. A. BENNETT, H. P. MANNING and H. E. SLAUGHT were appointed as the Committee on Publications beginning with the volume of the MONTHLY for 1922.

(f) A committee consisting of three Association members of the National Committee on Mathematical Requirements, Professors J. W. YOUNG, C. N.

MOORE and H. W. TYLER, was appointed for the purpose of coöperating with a similar committee, from the Society for the Promotion of Engineering Education, in an investigation of the teaching of mathematics in engineering schools.

(g) The Association received an invitation from the department of mathematics of Cornell University to hold its summer meeting at Ithaca in either of the years 1923 or 1924, in conjunction with the American Mathematical Society.

W. D. CAIRNS, *Secretary-Treasurer*.

MATHEMATICAL PROBLEMS IN THE WORK OF THE UNITED STATES COAST AND GEODETIC SURVEY.¹

By WALTER D. LAMBERT and OSCAR S. ADAMS,

Division of Geodesy, U. S. Coast and Geodetic Survey.

In the office of the United States Coast and Geodetic Survey there are three divisions that employ mathematics to a considerable extent in the routine work which they perform. These are the Division of Geodesy, the Division of Tides and Currents, and the Division of Terrestrial Magnetism. We shall aim to give some account of the nature and extent of the mathematics required as a tool in each of these three divisions.

The work of the Division of Geodesy is almost entirely mathematical and the problems that are presented for consideration are many and varied. The routine work consists of the computation and adjustment of triangulation, the computation of astronomical observations, the computation of gravity observations, and the computation and adjustment of observations in precise leveling. This is a bill of fare that should be sufficiently comprehensive to suit the taste of any one who is mathematically inclined (or should we say, in accordance with popular belief, mathematically deranged?)

The computation of triangulation and of astronomical observations requires a thorough training in trigonometry with especial attention to the use of logarithmic tables. This training should be directed toward the attainment of both speed and accuracy, with a considerable premium placed on accuracy. It may be thought that common sense would suffice for the use of tables, and so it would seem; but, if so, good common sense is oftentimes sadly lacking in men when they graduate from college. In the examinations given by the Coast and Geodetic Survey, mistakes are made by candidates that are so utterly absurd that one can hardly explain the psychology of the errors.

The adjustment of observations by the method of least squares forms the greater part of the purely mathematical work in the general routine of the geodetic division. This adjustment finds its most extensive field of application when applied to triangulation. The regular method employed in the Survey for this purpose is that of adjustment with condition equations as distinguished from

¹ Read at the meeting of the Ohio Section of the Mathematical Association of America, March 25, 1921.